

GRE optics review

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Light is an electromagnetic wave

$$E = E_0 \cos(kz - \omega t + \varphi)$$

k - wave vector; in the medium $k = \frac{2\pi n}{\lambda} = \frac{\omega n}{c} = \frac{2\pi f \cdot n}{c}$

Here ω - angular frequency [$\frac{\text{rad}}{\text{s}}$], f - frequency [Hz]

λ - wavelength $\lambda = \frac{2\pi c}{\omega}$, n - refractive index

Frequency of light does not change inside any medium,
wavelength become smaller $\lambda = \lambda_0/n$, light propagates
slower $v = c/n$

When speaking of light as a stream of photons

Energy of a photon $E_{ph} = \hbar\omega$

Momentum of a photon $\vec{P}_{ph} = \hbar\vec{k}$

Flux of photon is proportional to the light intensity ($|E_0|^2$)

$E(z,t) = E_0 \cos(kz - \omega t)$ running wave, propagating in $+z$ direction

Standing wave = superposition of two running waves in
opposite directions

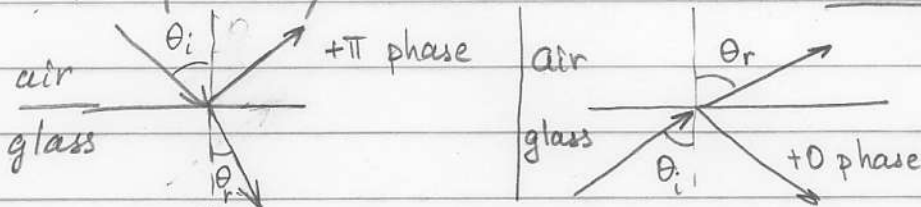
$$E_{st}(z,t) = \frac{1}{2} E_0 \cos(kz - \omega t) + \frac{1}{2} E_0 \cos(-kz - \omega t) = E_0 \cos kz \cos \omega t$$

Doppler effect - moving source/receiver $f = (1 - \frac{v_{s,r}}{c}) f_0$

Here $v_{s,r}$ is positive when the source moves away
from the receiver, and negative when they move
toward each other

Reflection & Refraction

1. When an electromagnetic wave (or any wave) reflects off a mirror, it acquires 180° (π) phase shift.
2. When a wave passes from a medium with low refractive index to one with high refractive index, it partially reflects with π phase shift.
3. When a wave passes from a medium with high refractive index to one with low refractive index, it partially reflects with no change in phase.



Snell's law $n \sin \theta = \text{const}$ (or $n_1 \sin \theta_i = n_2 \sin \theta_r$)

Since $\sin \theta \leq 1$, there is a critical angle $n_1 \sin \theta_c = n_2$

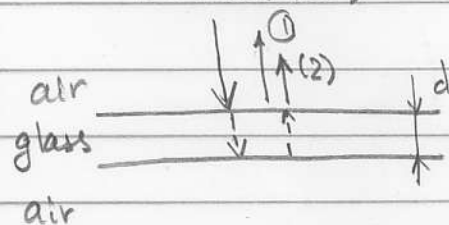
For this and any higher angle all light is reflected (Total internal reflection)

Interference: two waves interfere constructively if they are in phase: $\Delta \phi = 0, 2\pi, 4\pi, \dots$

They interfere destructively, if they are out of phase

$$\Delta \phi = \pi, 3\pi, 5\pi, \dots$$

Thin film interference (of two waves reflected off two boundaries)



The wave reflected off the bottom travelled extra distance $2d$, and acquired extra phase $[E \cos(kz - \omega t)]$

$$\Delta \phi_2 = k \cdot 2d = \frac{2\pi n}{\lambda_0} \cdot 2d$$

The wave reflected from the top received an additional π phase shift $\Delta\phi_1 = \pi$

Phase difference b/w two waves: $\frac{2\pi h}{\lambda_0} \cdot 2d - \pi$

Constructive interference: $\frac{2\pi h}{\lambda_0} \cdot 2d - \pi = 2\pi m \quad m = 0, 1, 2, \dots$
 or $2hd = (m + \frac{1}{2}) \lambda_0$

Destructive interference: $\frac{2\pi h}{\lambda_0} \cdot 2d - \pi = \pi + 2\pi m \quad m = 0, 1, 2, \dots$
 or $2hd = m \lambda_0$

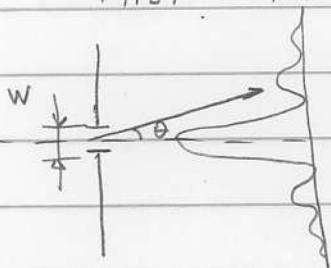
Single-slit diffraction, two-slit interference, diffraction gratings

General equation

$$I(\theta) = \left(\frac{\sin\left(\frac{\pi w}{\lambda} \sin\theta\right)}{\frac{\pi w}{\lambda} \sin\theta} \right)^2 \left(\frac{\sin\left(N \frac{\pi d}{\lambda} \sin\theta\right)}{\sin\left(\frac{\pi d}{\lambda} \sin\theta\right)} \right)^2$$

w - width of a slit, d - distance b/w two slits

First term - single slit diffraction



Diffraction maxima $\sin\left(\frac{\pi w}{\lambda} \sin\theta\right) = \pm 1$

$$\frac{\pi w}{\lambda} \sin\theta = \frac{\pi}{2} \pm \pi m \Rightarrow \boxed{w \sin\theta = (m + \frac{1}{2}) \lambda; \quad m = 0, 1, 2, \dots}$$

Diffraction minima $\sin\left(\frac{\pi w}{\lambda} \sin\theta\right) = 0$

$$\frac{\pi w}{\lambda} \sin\theta = \pi m \Rightarrow \boxed{w \sin\theta = m \lambda; \quad m = 1, 2, 3, \dots}$$

Diffraction grating ($N \gg 1$)

Diffraction orders correspond to global maxima $\Rightarrow \sin\left(\frac{\pi d}{\lambda} \sin\theta\right) = 0$
 or $\boxed{d \sin\theta = m \lambda; \quad m = 0, 1, 2, \dots}$

These conditions provide large ($\sim N^2$) peaks, separated by very low background

Special case: two-slit interference, $N=2$

Mathematics: $\sin 2x = 2 \sin x \cos x \Rightarrow \frac{\sin 2x}{\sin x} = 2 \cos x$

For $w \ll d$ (thin slits)

$$I(\theta) \propto \cos^2 \left(\frac{\pi d}{\lambda} \sin \theta \right)$$

Interference maxima: $\frac{\pi d}{\lambda} \sin \theta = \pi m$

$$\boxed{d \sin \theta = m \lambda}$$

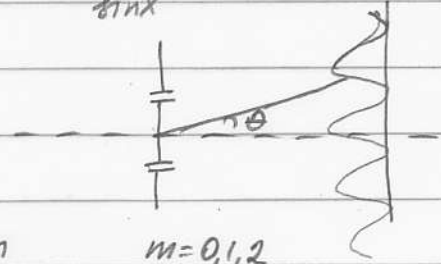
$m=0,1,2$

Interference minima:

$$\frac{\pi d}{\lambda} \sin \theta = \frac{\pi}{2} + \pi m$$

$m=0,1,2..$

$$\boxed{d \sin \theta = \left(m + \frac{1}{2}\right) \lambda}$$



Optical resolution

The diffraction of light on finite-size aperture of optics (lenses, mirrors) limits how well you can resolve spatial structure

Rayleigh's resolution criterion

Minimum resolvable angle: $\theta_{\min} = 1.22 \lambda/D$

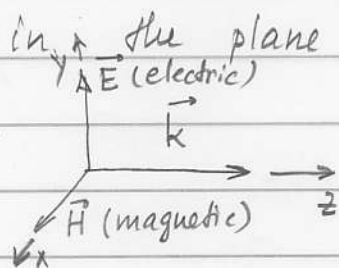
where D is the aperture size

[Note, that this criterion roughly corresponds to the measurement of the width of the first diffraction maxima:

$$\frac{\pi D}{\lambda} \sin \theta \approx \pi \Rightarrow \sin \theta \approx \theta \approx \lambda/D]$$

Polarization and polarizers

Polarization characterizes the direction of electric component of electro-magnetic vector. Polarization vector always lays in the plane orthogonal to the light propagation direction



Linear polarization: E-field oscillate along certain direction

$$\vec{E} = E_0 \cos \omega t \vec{e}_y$$

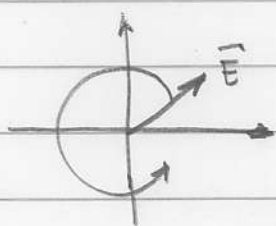
Polarizers let through a light polarized in particular direction (along the polarizer axis)



$$I_{out} = I_{in} \cos^2 d$$

where d is the angle b/w the direction of linear polarization and the polarizer axis.

Circular polarization: \vec{E} -field direction rotates in x-y plane



$$\vec{E} = \frac{1}{\sqrt{2}} E_0 \cos \omega t \vec{e}_x \pm \frac{1}{\sqrt{2}} E_0 \sin \omega t \vec{e}_y$$

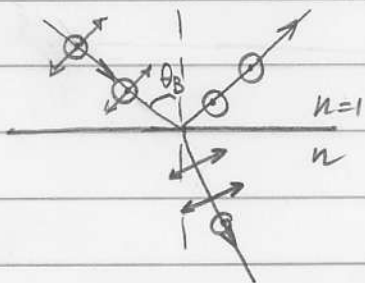
we can treat the circular polarization as a combination of two linear polarization, delayed by $\pi/2$ with respect to each other.

When circularly polarized light passes the polarizer, the output intensity is angle-independent and always $I_{out} = \frac{1}{2} I_{in}$

$$[\text{Indeed } I_{out} \propto \left(\frac{1}{\sqrt{2}} E_0\right)^2 \cos^2 d + \left(\frac{1}{\sqrt{2}} E_0\right)^2 \sin^2 d = \frac{1}{2} E_0^2]$$

Polarizer alone cannot distinguish circularly-polarized and unpolarized light.

Brewster's law

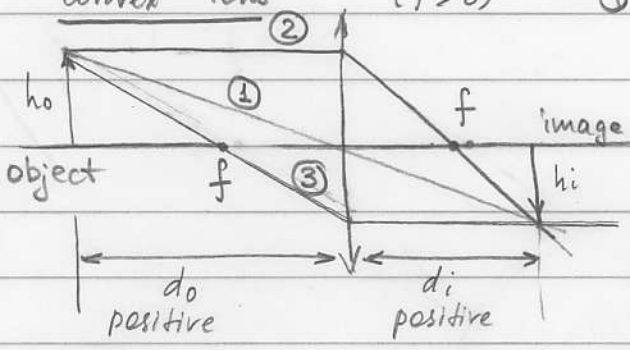


If light is incident at Brewster's angle $\tan \theta_B = n$, then reflected beam is polarized.

Geometrical optics.

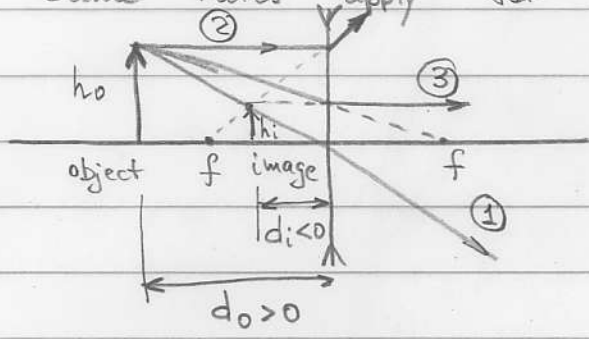
Convex lens ($f > 0$)

① A ray that goes through the center does not change direction.



② The ray that goes from infinity, goes through the focus after the lens.
 ③ The ray that goes through the focus, continues to infinity.

Same rules apply for concave lenses ($f < 0$)

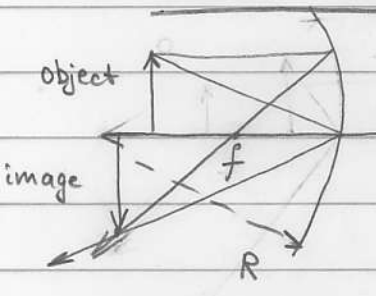


Lens formula

$$\frac{1}{d_i} + \frac{1}{d_o} = \frac{1}{f}$$

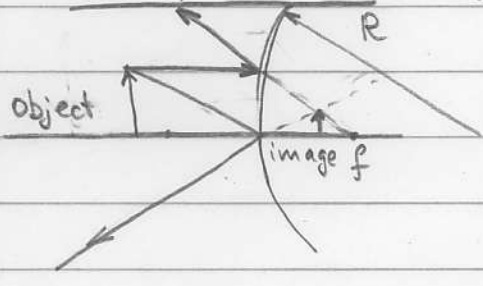
magnification $M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$

Concave mirror $f = R/2 > 0$

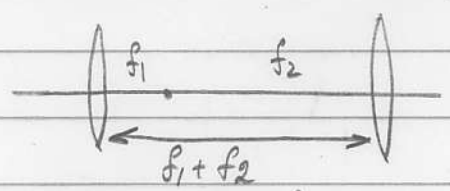


same rules work for mirrors as for the lenses, but now beams are reflected.

Convex mirror $f = -R/2 < 0$

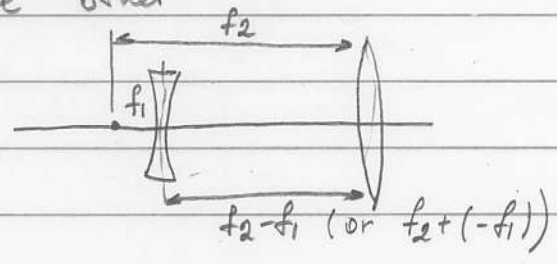


Telescope: two-lens assembly where the image for one lens becomes object for the other



magnification f_2/f_1

or



magnification f_2/f_1