GRE optics rewiew (September 2009)

Irina Novikova

Light is an electromagnetic wave

$E = E_0 \cos(kz - \omega t + \varphi)$

k – wave vector; in the medium $k = \frac{2\pi n}{\lambda} = \frac{\omega n}{c} = \frac{2\pi f \cdot n}{c}$

Here ω – angular frequency $[\frac{rad}{s}]$, f – frequency $[Hz]$

λ – wavelength $\lambda = \frac{2\pi c}{n\omega}$, n – refractive index

Frequency of light does not change inside any medium, wavelength become smaller $\lambda = \lambda_0/n$, light propagates slower $v = c/n$

When speaking of light as a stream of photons

Energy of a photon $E_{ph} = \hbar\omega$

Momentum of a photon $\vec{P}_{ph} = \hbar\vec{k}$

Flux of photon is proportional to the light intensity $(|E_0|^2)$

$E(z,t) = E_0 \cos(kz - \omega t)$ running wave, propagating in $+z$ direction

Standing wave = superposition of two running waves in opposite directions

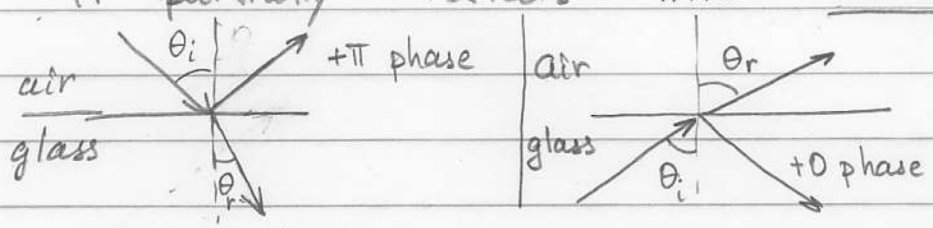
$E_{st}(z,t) = \frac{1}{2} E_0 \cos(kz - \omega t) + \frac{1}{2} E_0 \cos(-kz - \omega t) = E_0 \cos kz \cos \omega t$

Doppler effect – moving source/receiver $f = (1 - \frac{v_{s,r}}{c}) f_0$

Here $v_{s,r}$ is positive when the source moves away from the receiver, and negative when they move toward each other

Reflection & Refraction

1. When an electromagnetic wave (or any wave) reflects off a mirror, it acquires 180° (π) phase shift.
2. When a wave passes from a medium with low refractive index to one with high refractive index, it partially reflects with π phase shift.
3. When a wave passes from a medium with high refractive index to one with low refractive index, it partially reflects with no change in phase.



Snell's law $n \sin\theta = const$ (or $n_1 \sin\theta_i = n_2 \sin\theta_r$)

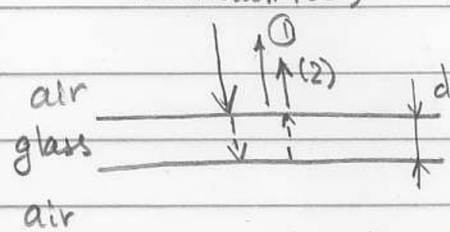
Since $\sin\theta \leq 1$, there is a critical angle $n_1 \sin\theta_i = n_2$

For this and any higher angle all light is reflected (Total internal reflection)

Interference: two waves interfere constructively if they are in phase: $\Delta\varphi = 0, 2\pi, 4\pi, \ldots$

They interfere destructively, if they are out of phase $\Delta\varphi = \pi, 3\pi, 5\pi \ldots$

Thin film Interference (of two waves reflected off two boundaries)



The wave reflected off the bottom travelled extra distance 2d, and acquired extra phase $[E \cos(kz - \omega t)]$

$\Delta\varphi_2 = k \cdot 2d = \frac{2\pi n}{\lambda_0} \cdot 2d.$

The wave refrected from thee top received an additional π phase shift $\Delta\varphi_1 = \pi$

Phase difference b/w two waves: $\frac{2\pi n}{\lambda_0} \cdot 2d - \pi$

Constructive interference: $\frac{2\pi n}{\lambda_0} \cdot 2d - \pi = 2\pi m \qquad m = 0, 1, 2 \ldots$

or $2nd = (m + \frac{1}{2}) \lambda_0$

Destructive interference: $\frac{2\pi n}{\lambda_0} \cdot 2d - \pi = \pi + 2\pi m \qquad m = 0, 1, 2 \ldots$

or $2nd = m \lambda_0$

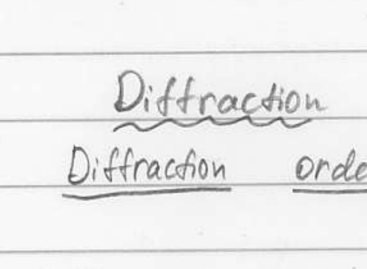
Single - slit diffraction, two-slit interference, diffraction gratings

General equation

$$I(\theta) = \left(\frac{\sin\left(\frac{\pi w}{\lambda} \sin\theta\right)}{\frac{\pi w}{\lambda} \sin\theta} \right)^2 \left(\frac{\sin\left(N \frac{\pi d}{\lambda} \sin\theta\right)}{\sin\left(\frac{\pi d}{\lambda} \sin\theta\right)} \right)^2$$

w - width of a slit, d - distance b/w two slits

First term - single slit diffraction



Diffraction maxima $\sin\left(\frac{\pi w}{\lambda} \sin\theta\right) = \pm 1$

$\frac{\pi w}{\lambda} \sin\theta = \frac{\pi}{2} \pm \pi m \Rightarrow$ $\boxed{w \sin\theta = (m + \frac{1}{2}) \lambda; \; m = 0, 1, 2 \ldots}$

Diffraction minima $\sin\left(\frac{\pi w}{\lambda} \sin\theta\right) = 0$

$\frac{\pi w}{\lambda} \sin\theta = \pi m \Rightarrow$ $\boxed{w \sin\theta = m\lambda; \quad m = 1, 2, 3 \ldots}$

Diffraction grating $(N \gg 1)$

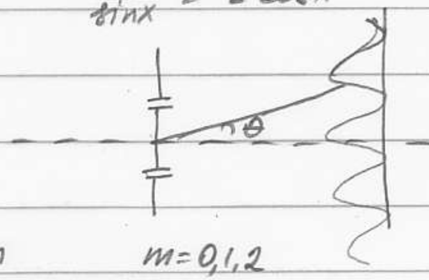
Diffraction orders correspond to global maxima $\Rightarrow \sin\left(\frac{\pi d}{\lambda} \sin\theta\right) = 0$

or $\boxed{d \sin\theta = m\lambda; \quad m = 0, 1, 2 \ldots}$

These conditions provide large $(\sim N^2)$ peaks, separated by very low background

Special case: two-slit interference, $N=2$

Mathematics: $\sin 2x = 2\sin x \cos x \Rightarrow \frac{\sin 2x}{\sin x} = 2\cos x$



For $w \ll d$ (thin slits)

$$I(\theta) \propto \cos^2\left(\frac{\pi d}{\lambda}\sin\theta\right)$$

Interference maxima: $\frac{\pi d}{\lambda}\sin\theta = \pi m$ $\quad m = 0, 1, 2$

$$\boxed{d\sin\theta = m\lambda}$$

Interference minima: $\frac{\pi d}{\lambda}\sin\theta = \frac{\pi}{2} + \pi m$ $\quad m = 0, 1, 2..$

$$\boxed{d\sin\theta = (m + \tfrac{1}{2})\lambda}$$

Optical resolution

The diffraction of light on finite-size aperture of optics (lenses, mirrors) limits how well you can resolve spatial structure

Rayleigh's resolution criterion

Minimum resolvable angle: $\theta_{min} = 1.22\,\lambda/D$

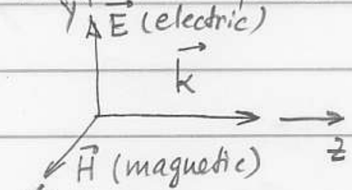
where D is the aperture size

[Note, that this criterion roughly corresponds to the measurement of the width of the first diffraction maxima:

$\frac{\pi D}{\lambda}\sin\theta \approx \pi \Rightarrow \sin\theta \approx \theta \approx \lambda/D$]

Polarization and polarizers

Polarization characterizes the direction of electric component of electro-magnetic vector. Polarization vector always lays in the plane orthogonal to the light propagation direction



<u>Linear</u> <u>polarization</u>: E-field oscillate along certain direction

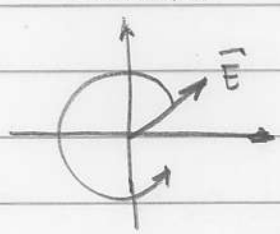
$$\vec{E} = E_0 \cos\omega t \ \vec{e}_y$$

<u>Polarizers</u> let through a light polalized in particular direction (along the polarizer axis)

$$\boxed{I_{out} = I_{in} \cos^2\alpha}$$

where α is the angle b/w the direction of linear polarization and the polarizer axis.

Circular polarization: $\vec{E}$ - field direction rotates in x-y plane



$$\vec{E} = \frac{1}{\sqrt{2}} E_0 \cos\omega t \ \vec{e}_x \pm \frac{1}{\sqrt{2}} E_0 \sin\omega t \ \vec{e}_x$$

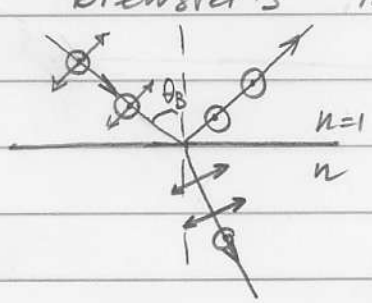
we can treat the circular polarization as a combination of two linear polarization, delayed by $\pi/2$ with respect to each other

When circularly polarized light passes the polarizer, the output intensity is angle-independent and always $I_{out} = \frac{1}{2} I_{in}$

[Indeed $I_{out} \propto \left(\frac{1}{\sqrt{2}} E_0\right)^2 \cos^2\alpha + \left(\frac{1}{\sqrt{2}} E_0\right)^2 \sin^2\alpha = \frac{1}{2} E_0^2$]

Polarizer alone <u>cannot distinguish</u> circularly-polarized and unpolarized light.

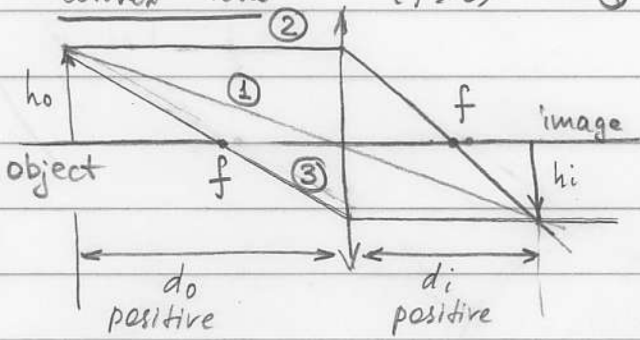
Brewster's law



If light is incident at Brewster's angle $\tan\theta_B = n$, then reflected beam is polarized.

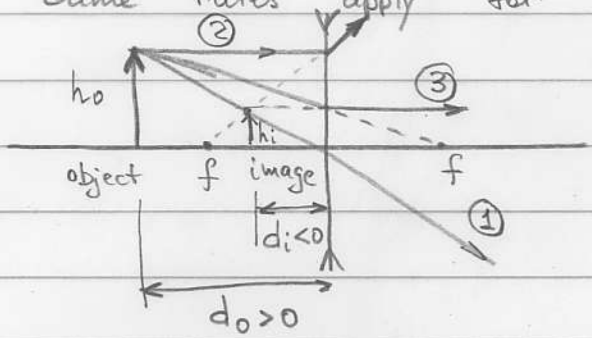
Geometrical optics.

Convex lens $(f>0)$



① A ray that goes through the center does not change direction.
② The ray that goes from infinity, goes through the focus after the lens.
③ The ray that goes through the focus, continues to infinity.

Same rules apply for concave lenses $(f<0)$

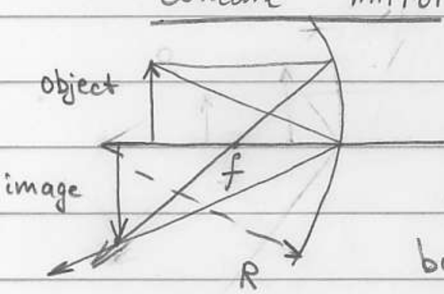


Lens formula

$$\frac{1}{d_i} + \frac{1}{d_o} = \frac{1}{f}$$

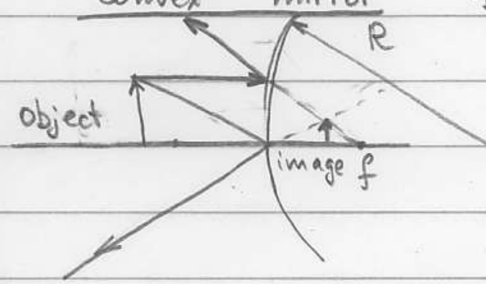
magnification $M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$

Concave mirror $f = R/2 > 0$

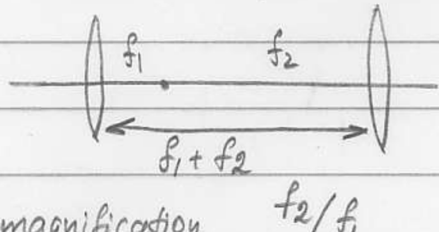


same rules work for mirrors as for the lenses, but now beams are reflected.

Convex mirror $f = -R/2 < 0$

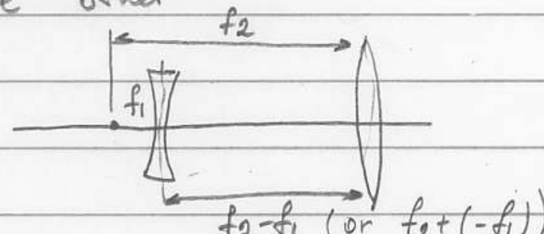


Telescope : two-lens assembly where the image for one lens becomes object for the other



magnification f_2/f_1

or



magnification f_2/f_1