35. **Light of wavelength 5200 angstroms is incident normally on a transmission diffraction grating with 2000 lines per centimeter. The first-order diffraction maximum is at an angle, with respect to the incident beam, that is most nearly**

(A) 3°
(B) 6°
(C) 9°
(D) 12°
(E) 15°

Correct answer is (B)

If you remember diffraction grating equation

$$I \propto \underbrace{\left(\frac{\sin\left(\frac{\pi W}{\lambda}\sin\theta\right)}{\left(\frac{\pi W}{\lambda}\sin\theta\right)} \right)^2}_{\text{from one slit (not relevant here)}} \underbrace{\left(\frac{\sin\left(N\frac{\pi d}{\lambda}\sin\theta\right)}{\sin\left(\frac{\pi d}{\lambda}\sin\theta\right)} \right)^2}_{\text{from N slits.}}$$

Diffraction maxima => denomitator of the second fraction is zero

$\frac{\pi d}{\lambda}\sin\theta = \pi m$ m - diffraction order

for the first order $m=1$

$$\sin\theta = \frac{\pi\lambda}{\pi d} = \frac{\lambda}{d} = \frac{5200 \cdot 10^{-10}\,m}{\left(10^{-2}\,m/2000\right)} = 0.104\ rad = 6^\circ$$

20. In a double-slit interference experiment, d is the distance between the centers of the slits and w is the width of each slit, as shown in the figure above. For incident plane waves, an interference maximum on a distant screen will be "missing" when

(A) $d = \sqrt{2}w$
(B) $d = \sqrt{3}w$
(C) $2d = w$
(D) $2d = 3w$
(E) $3d = 2w$

The correct answer is (D)

For two-slit interference

$$I \propto \underbrace{\left(\frac{\sin\left(\frac{\pi w}{\lambda}\sin\theta\right)}{\left(\frac{\pi w}{\lambda}\sin\theta\right)} \right)^2}_{\text{one slit}} \underbrace{\left(\frac{\sin\left(2\frac{\pi d}{\lambda}\sin\theta\right)}{\sin\left(\frac{\pi d}{\lambda}\sin\theta\right)} \right)^2}_{\text{two slits}} =$$

$$= \left(\frac{\sin\left(\frac{\pi w}{\lambda}\sin\theta\right)}{\left(\frac{\pi w}{\lambda}\sin\theta\right)} \right)^2 \cdot 4\cos^2\left(\frac{\pi d}{\lambda}\sin\theta\right)$$

One of the interference maxima is missing when it overlaps with diffraction minimum.

Interference $\underline{\text{maxima}} \Rightarrow \cos\left(\frac{\pi d}{\lambda}\sin\theta\right) = \pm 1 \qquad \frac{\pi d}{\lambda}\sin\theta = \pi m \quad m = 0, 1, 2 \ldots$

Diffraction $\underline{\text{minima}} \Rightarrow \sin\left(\frac{\pi w}{\lambda}\sin\theta\right) = 0 \qquad \frac{\pi w}{\lambda}\sin\theta = \pi n \quad n = 1, 2, 3$

For these two conditions to be true at the same time

$\frac{w}{d} = \frac{n}{m}$, and we also keep in mind that $w < d$

The only acceptable answer then is (D)

$m = 2,\ n = 3 \qquad \underline{3w = 2n}$

59. The approximate number of photons in a femtosecond (10^{-15}s) pulse of 600 nanometers wavelength light from a 10-kilowatt peak-power dye laser is

(A) 10^3
(B) 10^7
(C) 10^{11}
(D) 10^{15}
(E) 10^{18}

60. The Lyman alpha spectral line of hydrogen (λ = 122 nanometers) differs by 1.8×10^{-12} meter in spectra taken at opposite ends of the Sun's equator. What is the speed of a particle on the equator due to the Sun's rotation, in kilometers per second?

(A) 0.22
(B) 2.2
(C) 22
(D) 220
(E) 2200

82. Consider two horizontal glass plates with a thin film of air between them. For what values of the thickness of the film of air will the film, as seen by reflected light, appear bright if it is illuminated normally from above by blue light of wavelength 488 nanometers?

(A) 0, 122 nm, 244 nm
(B) 0, 122 nm, 366 nm
(C) 0, 244 nm, 488 nm
(D) 122 nm, 244 nm, 366 nm
(E) 122 nm, 366 nm, 610 nm

82. Bright → constructive interference

Top boundary (glass-air) - no phase shift
Bottom boundary (air-glass) — π phase shift

Total shift $-\pi + \frac{2\pi}{\lambda}\cdot 2d = 2\pi \cdot \underset{\uparrow}{m}$ $m=0,1,2.$

for constructive interference

$$d = \left(m+\tfrac{1}{2}\right)\cdot\frac{\lambda}{2} = \left(m+\tfrac{1}{2}\right)\cdot 244\text{ nm}$$

The correct answer is (E)

59. Energy of one photon

$$E_{ph} = \hbar\omega = \hbar\frac{2\pi c}{\lambda} = \frac{2\pi\cdot10^{-34}\cdot3\cdot10^{8}}{6\cdot10^{-7}}$$

$$\approx 3\cdot10^{-19}\text{ J}$$

Energy of a pulse

$$E_{pulse} = 10^4\text{ W}\cdot10^{-15}\text{ s} = 10^{-11}\text{ J}$$

# of photons $\frac{E_{pulse}}{E_{ph}} = \frac{10^{-11}\text{ J}}{3\cdot10^{-19}\text{ J}} = 3\cdot10^{7}$

Correct answer ic (B)

60. Doppler effect

$$f_{1,2} = \left(1 \pm \frac{V_p}{c}\right) f_0$$

$$\Delta f = \frac{2V_p}{c} f_0$$

$$\Delta\lambda = \lambda_1 - \lambda_2 = \left(\frac{c}{f_1} - \frac{c}{f_2}\right) =$$

$$= -\frac{c\,\Delta f}{f_1\cdot f_2} \approx -\frac{c}{f_0}\frac{\Delta f}{f_0} = -\lambda\frac{\Delta f}{f_0}$$

thus $\left|\frac{\Delta\lambda}{\lambda}\right| \approx \left|\frac{\Delta f}{f_0}\right|$

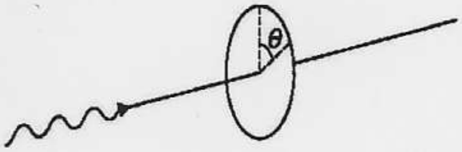
$$\frac{\Delta\lambda}{\lambda} = \frac{2V_p}{c}$$

$$V_p = \frac{3\cdot10^8\text{ m/s}}{2}\cdot\frac{1.8\cdot10^{-12}\text{ m}}{1.22\cdot10^{-7}\text{ m}}$$

$$= \left\{\begin{matrix}\text{some}\\ \text{coefficient} \sim 2\end{matrix}\right\}\cdot 10^3\text{ m/s}$$

Thus, the correct answer is (B)

(since it asks for v in km/s)



67. A steady beam of light is normally incident on a piece of polaroid. As the polaroid is rotated around the beam axis, the transmitted intensity varies as $A + B\cos 2\theta$, where θ is the angle of rotation, and A and B are constants with $A > B > 0$. Which of the following may be correctly concluded about the incident light?

(A) The light is completely unpolarized.
(B) The light is completely plane polarized.
(C) The light is partly plane polarized and partly unpolarized.
(D) The light is partly circularly polarized and partly unpolarized.
(E) The light is completely circularly polarized.

The correct answer is (C)

Unpolarized and circularly polarized light give constant intensity after a polarizer. $I_{out} = \frac{1}{2} I_{in}$

Linearly polarized light gives $I_{out} = I_{in} \cos^2\theta$

Thus, total transmission through a polarizer is

$$I_{total} = \frac{1}{2} I_{unpol} + \frac{1}{2} I_{circ} + I_{lin} \cos^2\theta$$

recalling $\cos^2\theta = \frac{1}{2}(\cos 2\theta - 1)$

$$I_{total} = \underbrace{\left[\frac{1}{2} I_{unpol} + \frac{1}{2} I_{circ} - \frac{1}{2} I_{lin}\right]}_{A} + \underbrace{\frac{1}{2} I_{lin}}_{B} \cos 2\theta$$

58. A collimated laser beam emerging from a commercial HeNe laser has a diameter of about 1 millimeter. In order to convert this beam into a well-collimated beam of diameter 10 millimeters, two convex lenses are to be used. The first lens is of focal length 1.5 centimeters and is to be mounted at the output of the laser. What is the focal length, f, of the second lens and how far from the first lens should it be placed?

	f	Distance
(A)	4.5 cm	6.0 cm
(B)	10 cm	10 cm
(C)	10 cm	11.5 cm
(D)	15 cm	15 cm
(E)	15 cm	16.5 cm

To convert the beam of ⌀1 mm to ⌀10 mm, the telescope should have magnification 10. $f_1 = 1.5\,cm$

$$\frac{f_2}{f_1} = 10 \quad \Rightarrow \quad f_2 = 15\,cm$$

The distance b/w two lenses

$$f_1 + f_2 = 16.5\,cm$$

Correct answer is (E)

68. The angular separation of the two components of a double star is 8 microradians, and the light from the double star has a wavelength of 5500 angstroms. The smallest diameter of a telescope mirror that will resolve the double star is most nearly

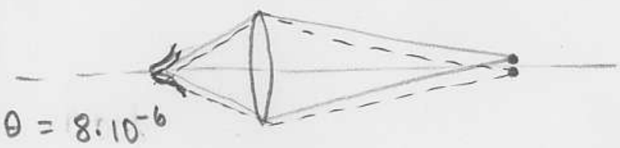
(A) 1 mm
(B) 1 cm
(C) 10 cm
(D) 1 m
(E) 100 m

Angular resolution is limited by the diffraction at the limited telescone / lens. aperture

Rayleigh criterion:

$\sin\theta = 1.22 \frac{\lambda}{d}$ [It is probably safe to just remember $\sin\theta \sim \lambda/d$]

$d = 1.22 \frac{\lambda}{\sin\theta}$



For small angle $\sin\theta \approx \theta = 8 \cdot 10^{-6}$

$$d = 1.22 \cdot \frac{5.5 \cdot 10^{-7}\,m}{8 \cdot 10^{-6}} \sim 0.1m$$

Correct answer is (C)

21. A soap film with index of refraction greater than air is formed on a circular wire frame that is held in a vertical plane. The film is viewed by reflected light from a white-light source. Bands of color are observed at the lower parts of the soap film, but the area near the top appears black. A correct explanation for this phenomenon would involve which of the following?

I. The top of the soap film absorbs all of the light incident on it; none is transmitted.

← wrong! the film does not absorb light at all

II. The thickness of the top part of the soap film has become much less than a wavelength of visible light.

← True! The top appears black since all light is transmitted

III. There is a phase change of 180° for all wavelengths of light reflected from the front surface of the soap film.

IV. There is no phase change for any wavelength of light reflected from the back surface of the soap film.

Always true, since $n_{soap} > n_{air}$

(A) I only
(B) II and III only
(C) III and IV only
(D) I, II, and III
(E) II, III, and IV

Correct answer is (E)

22 A simple telescope consists of two convex lenses, the objective and the eyepiece, which have a common focal point P, as shown in the figure above. If the focal length of the objective is 1.0 meter and the angular magnification of the telescope is 10, what is the optical path length between objective and eyepiece?

(A) 0.1 m
(B) 0.9 m
(C) 1.0 m
(D) 1.1 m
(E) 10 m

Since the angular magnification is 10, then

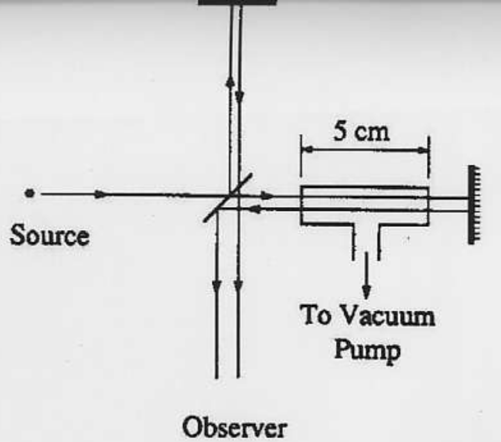
$$\frac{f_{objective}}{f_{eyepiece}} = 10$$

$$f_{eyepiece} = \frac{1}{10} f_{objective} = 0.1 m$$

Optical path = $f_{objective} + f_{eyepiece} =$

$= 1.1 m$

Correct answer is (D)



96. A gas-filled cell of length 5 centimeters is inserted in one arm of a Michelson interferometer, as shown in the figure above. The interferometer is illuminated by light of wavelength 500 nanometers. As the gas is evacuated from the cell, 40 fringes cross a point in the field of view. The refractive index of this gas is most nearly

(A) 1.02
(B) 1.002
(C) 1.0002
(D) 1.00002
(E) 0.98

Fringe condition → difference in optical paths is integer number of 2π

Optical path defined as kL, where $k = \frac{2\pi n}{\lambda}$ is a wave vector

With filled cell the optical path is: $\overset{\text{double-pass}}{2} \cdot \frac{2\pi n_{air}}{\lambda} L_{cell}$ + everything else

With evacuated cell the optical path is: $2 \cdot \frac{2\pi}{\lambda} L_{cell}$ + everything else

($n_{vacuum} = 1$)

40 fringes crossed the screen → optical path changed by $40 \cdot 2\pi$

$$2 \cdot \frac{\cancel{2\pi}(n_{air} - 1)}{\lambda} L_{cell} = 40 \cdot \cancel{2\pi}$$

$$n_{air} = 1 + \frac{40\lambda}{L_{cell}} = 1 + \frac{20 \cdot 5 \cdot 10^{-7} m}{5 \cdot 10^{-2} m} = 1 + 2 \cdot 10^{-4} = 1.0002$$

The correct answer is (C)