Thus each final bubble has "cleaned out" most of the excess air in a region having a diameter about 10 times the final bubble radius. For my very crudely measured value $a_0 = 2\times10^{-3}$ cm, the radius $R_0 = L_0/2$ of the cleaned out region is then $R_0 = 5a_0 = 10^{-2}$ cm. How long does it take to clean out this region? Call t_0 the time it takes an air molecule to "explore" by diffusion a region of radius R_0. According to Eq. (13) that time is given by

$$t_0 = R_0^2/\lambda c. \tag{21}$$

The chance that during the exploration of this region the air molecule will be captured in the bubble should be proportional to the time spent in the bubble volume; that time should be proportional to the bubble volume. Thus without worrying about factors of two (the bubble volume is not constant) we multiply Eq. (21) by the volume ratio R_0^3/a_0^3. That gives our estimated clean-out time t:

$$t = R_0^5/a_0^3\lambda c. \tag{22}$$

Since the ratio R_0/a_0 is fairly well known (it depends only on our fairly well measured value of f_a) but a_0 is poorly measured, we put in our value $R_0/a_0 = 5$ and write Eq. (22) as $t = 5^5a_0^2/\lambda c$. For the mean free path λ of the air molecule diffusing in water we take the edge length of the cube occupied by one water molecule in the liquid: $\lambda = 3\times10^{-8}$ cm. Taking $c = 300$ m/sec (thermal velocity), and our crude value $a_0 = 2\times10^{-3}$ cm, we find an estimated bubble growth time

$$t = 5^5(2\times10^{-3})^2/(3\times10^{-8})(3\times10^4) = 14 \text{ sec}. \tag{23}$$

Because of the large uncertainty in a_0, the better-than-order-of-magnitude agreement of Eq. (23) with my observations has to be pure luck. But the order-of-magnitude agreement is not, and supports the model with practically constant number of bubbles, each growing larger with time because of the capture of diffusing air molecules.

Finally, what is it that determines the initial number of bubbles per unit volume, N_v? (That is what determines the final bubble radius a_0.) I believe there is a very simple concept that would enable me to predict N_v. But I haven't found the concept.

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Measurement of spectral line splittings with a scanning, student-grade, Fabry–Perot interferometer

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A simple method allowing precise measurement of spectral line splittings with a scanning, student-grade, Fabry–Perot interferometer is described. The isotope split between hydrogen and deuterium emission lines is taken as a characteristic experiment. Typical data are presented and analyzed. The experiment is suitable for a junior–senior-level laboratory.

The measurement of spectral line shapes and fine structure by interferometric techniques is a well-developed discipline.[1] In student experiments one usually uses a Fabry–Perot etalon where the mirror spacing is fixed. In this case one photographs the resulting ring pattern for later detailed analysis.[2] It is also possible, however, to make the same measurements with a "scanning" Fabry–Perot, that is a Fabry–Perot one of whose mirrors can be translated.

Research-grade interferometers of this type typically rely on piezoelectric translators or pressure changes to provide the necessary tuning.[3–6] Student-grade versions of the scanning Fabry–Perot, using a fine-screw translator, are common to most college laboratories. Frequently they use the base and movable carriage of a Michelson interferometer for stability and tuning.[7] The experiment described below is easily performed using such an interferometer and is suitable for a junior–senior-level laboratory.

The usual method for determining very small splittings with a scanning Fabry–Perot is to vary the mirror separation l by two or three wavelengths while monitoring the transmitted intensity. For normal incidence, plane waves, and an air-spaced Fabry–Perot, the ratio of transmitted to incident intensity is given by Born and Wolf[8] as

$$I_t/I_i = [1 + F\sin^2(\delta/2)]^{-1},$$

where $\delta = 4\pi l/\lambda$ and $F = 4R/(1-R)^2$, R being the intensity reflection coefficient, F the *coefficient of finesse*, and λ the wavelength of the incident light. Obviously the transmitted intensity is a maximum whenever $\delta = 2\pi n$ or when $l = n\lambda/2$, where n is a positive integer. Clearly, if two wavelengths are present, then two peaks occur in the transmitted intensity, namely, when δ satisfies the above condition for each wavelength. The *free spectral range* (FSR) is defined as that spectral range that can be scanned before encountering the same wavelength at a higher order. In practice one assumes that the FSR is constant and is given by the mirror separation through $\nu_{\rm FSR} = c/2l$ or $\lambda_{\rm FSR} = \lambda_0^2/2l$. One thereby is provided with a frequency scale as the intensity pattern repeats itself every free spectral range. The splitting of spectral lines can then be measured in terms of the known FSR. This technique obviously fails if the mirror movement Δl is no longer a small fraction of the total length l because the FSR is no longer constant. The method also demands that the mirror translation be done with great precision.

There is a conflicting criterion if one needs a very large free spectral range, since then the mirror separation must be relatively small. As an example suppose one wishes to measure the 1.8-Å separation between the Hα spectral lines of hydrogen and deuterium. If we demand a FSR of 5 Å to allow clear resolution of the spectrum, then the separation must be $l = 4\times10^{-2}$ cm, which is quite small although still approximately 10^3 wavelengths. A second difficulty is the need to translate precisely one mirror by a small fraction of a wavelength. If one desires a resolution of $\frac{1}{10}$ of an FSR then $\Delta l = \lambda/20$ is needed. Although this criterion is easily met by piezoelectric drivers the usual student interferometer is unable to provide the necessary precision. For these reasons, the technique I describe below is advantageous.

The principle of the experiment is elementary. Begin with the mirrors as close together as possible. Slowly separate the mirrors. The initially unresolved splitting becomes visible as a second set of rings close to the first. As the mirrors continue to separate the pair of rings move farther apart. At a certain mirror separation l_1 the outer ring of the inner pair overtakes the inner ring of the outer pair and one sees only a single set of rings. At this point the mirror separation l_1 corresponds to one free spectral range, $\lambda_1^{\rm FSR} = \lambda_0^2/2l_1$. The FSR is now exactly equal to the H–D splitting. If one is viewing the Balmer α line, then from the above equation $l_1 = 0.12$ cm. Translations of this size are

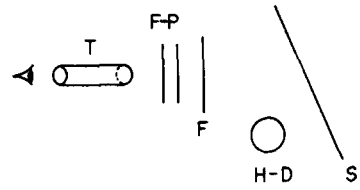

Fig. 1. Experimental setup consists of a hydrogen–deuterium discharge, H–D; screen, S; Fabry–Perot interferometer, F–P; a filter, F; and an optional telescope focused at infinity, T. Fringes are viewed through the Fabry–Perot using the screen as an extended source. A glass filter (Schott 590) was used to isolate the Hα line at 6565 Å.

easily achievable with great precision on a student interferometer. Frequently the point of zero mirror separation cannot be determined easily. If one continues to separate the mirrors through several free spectral ranges then the separation l_i at each overlap can be plotted (Fig. 2) and the slope analyzed to yield an accurate value for the spectral line splitting without knowledge of the zero beforehand.

To see these results explicitly we begin with the result for ring radii as given for example by Born and Wolf.[8] The radius of the pth ring is

$$r_p = f(\lambda/l)(p - 1 + \epsilon)^{1/2},$$

where f is the focal length of the lens that is used to image the rings (otherwise localized at infinity), λ is the wavelength, l is the mirror spacing, and ϵ $(0<\epsilon<1)$ takes account of fraction orders. We wish to determine the separation l_1, which will give $r_p = r_{p+1}$ for wavelengths λ and $\lambda + \Delta\lambda$. Using the equations for r_p and r_{p+1} and noting that l and f are necessarily identical for a single apparatus, yields

$$(\lambda + \Delta\lambda)(p - 1 + \epsilon) = \lambda(p + \epsilon).$$

Let $p + \epsilon = x$, which is the mirror separation in half wavelengths. Solving for x gives

$$x = \lambda/\Delta\lambda + 1.$$

If $\Delta\lambda = 1.8$ Å and $\lambda = 6565$ Å, then $x = 3648.2$ and $l_1 = 0.120$ cm, which is the result derived earlier. We can proceed entirely analogously for larger-order differences or in general when

$$r_p = r_{p+n}.$$

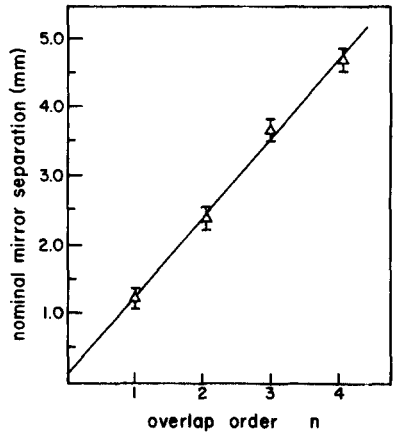

Fig. 2. Plot of typical data for the H–D isotope shift. Determination of the slope gives $\Delta l/\Delta n = 0.119$ cm. This corresponds to an isotope shift of 1.81 Å in good agreement with the actual value.

In this case, the separation is given by

$$x = n\lambda/\Delta\lambda + 1 .$$

This procedure is somewhat similar to that described by Ditchburn.[9]

Figure 1 shows a schematic of the apparatus used to measure the hydrogen–deuterium (H–D) isotope shift. A plot of a typical data set is shown in Fig. 2. The principle uncertainty in the experiment is due to the low finesse of the Fabry–Perot used. The intensity reflection coefficients were $R = 0.8$ yielding a *coefficient of finesse*, $F = 80$. This in turn means that the *finesse*

$$\mathcal{F} = \mathrm{FSR}/\gamma = 14 ,$$

where γ is the *bandwidth* of the interferometer for transmitted light. Therefore if the FSR = 5 Å, the bandwidth γ is 0.36 Å. At the first overlap the FSR is 1.8 Å and $\gamma = 0.13$ Å. This, when combined with the pressure broadened width of the spectral lines, leads to an uncertainty in the determination of the mirror separation at overlap of about ± 0.015 cm. By measuring the mirror separation at successive overlaps, a more accurate determination of the isotope shift can be accomplished. The result of the above measurement was $\Delta\lambda = 1.81 \pm 0.05$ Å.

Another special feature of this experiment is the extraordinary beauty of the ring patterns produced. With a conventional H–D discharge, besides the strong red rings, blue (4863 Å) and violet (4342 Å) rings are visible. When measuring the Balmer α splitting at 6565 Å an inexpensive Schott glass, cutoff filter is placed between the source and the Fabry–Perot. Without the filter all three sets are seen, each of which is itself a doublet that can be resolved.

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Fluctuations in light beams: A simple student experiment

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Light emitted from a thermal source has a fluctuating intensity; these fluctuations are a direct illustration of the elementary principles of statistical and quantum physics. We describe here a student experiment demonstrating these ideas. First the probability density for the intensity of thermal light inhabiting s coherence volumes is derived, using the Bose–Einstein distribution for photons in phase space. Then a simple method is shown for measuring this probability density using a quasithermal light source and a multichannel analyzer. Data are presented and a scheme for fitting the data to the theory is outlined.

I. INTRODUCTION

Light emitted from a thermal source fluctuates both spatially and temporally. The *spatial* fluctuations are due to the finite size of the source and the spatial incoherence of the photons emitted from various parts of the source.[1] *Temporal* fluctuations are due to the wide blackbody spectrum emitted by such a source; this in turn is a direct result of the Bose–Einstein statistics obeyed by photons.[2] The task of calculating and measuring the statistical quantities associated with these fluctuations has long been an important research problem[2,3]; for the case of thermal sources the physics is now well understood.

An important step forward both in helping to visualize the statistical behavior of thermal light and in obtaining actual measurements of the statistical quantities has been the discovery of the so-called quasithermal light source.[4] This source is, in fact, nothing more than a rotating ground-glass disk illuminated by a laser. The slowly evolving speckle pattern produced by such a source is statistically identical to that of a true thermal source except that: (i) The coherence times are much longer and hence easily measurable. (True thermal sources have coherence times $\lesssim 10^{-9}$ sec.) (ii) The number of photons associated with a coherence volume is $\gtrsim 10^{8}$ while that for a thermal source is $\lesssim 10^{-2}$. Thus with the quasithermal source we can measure the intensity directly. With a true thermal source one can only infer the intensity statistics from the photon